

Lecture 8

Big-Bang Nucleosynthesis

- Review (and future plan)
- BBN
 - general picture
 - ${}^4\text{He}$ abundance (equilibrium)
 - D abundance (quite reactions rate)

Review

1

CMB:

Saha's equation

$$\frac{n_e n_p}{n_H} = \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-\frac{I}{T}}$$

$$\frac{\partial n}{\partial t} + 3H \frac{\partial n}{\partial p} = \langle \sigma n \nu \rangle n$$

$\Gamma_{N \rightarrow n}$

$e\gamma \leftrightarrow e\gamma$ process freezes out

$$T = \frac{I}{2.27} = 0.25 \text{ eV} \quad [\text{electrons non-rel.}]$$

Neutrinos

$e\nu \leftrightarrow e\nu$ process freezes out

$$T_\nu = (G_F^2 M_0)^{-1/3} \sim 2 \text{ MeV} \quad [\text{neutrinos rel.}]$$

e^+e^- annihilation heats up Neutrinos

$$T_{\nu,0} \approx 2 \text{ K} \quad (T_{\gamma,0} = 2.6 \text{ K})$$

\leadsto cosmological bound on n_ν

Future Plans:

- Baryogenesis and Dark Matter
- Theory of perturbations (5 lectures)
 - \leadsto CMB Rubakov, Gorbunov II
 - \leadsto Large scale structure
 - \leadsto Inflation

BBN

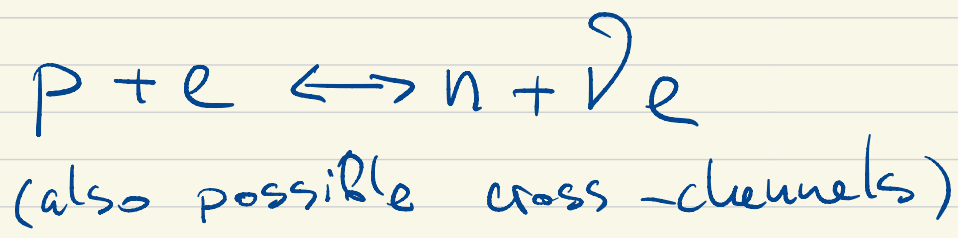
3

- BBN is the process of formation of light nuclei (H, He, D, Li, \dots) from p and n in the primordial plasma. It occurs at $T \sim 1 - 0.1 \text{ MeV}$. Atoms form much later (during recombination) and all heavier elements even later in stars.
- As a result of BBN we get 75% of H , 25% of He and $\sim 2 \cdot 10^{-5}$ of D . These abundances are measured and since they depend significantly on various parameters N_{eff} , η_B , etc., these parameters get constrained.

4
Neutron - proton ratio, freeze-out
of neutrons

Remember that $m_p \sim m_n \sim 1 \text{ GeV}$,
while $m_n - m_p = 1.3 \text{ MeV}$.

Let us check when the reaction



will go out of equilibrium

It is a weak interaction, so
 T^* will be close to that of
neutrinos

$$T^* \sim \left(\frac{M_0}{G_F^2} \right)^{1/3} \sim 2 \text{ MeV}$$

[A more precise calculation gives

$$T_n^* \sim 0.8 \text{ MeV}]$$

5

Both are comparable to $n_n - n_p$
[It is an important coincidence!]

Let us compute the abundances, assuming thermal equilibrium (ν 's just froze out, but they are still in the thermal eq.)

We have around n_p, e^\pm, ν 's


$$n_n \approx \exp\left[-\frac{m_n - \mu_n}{T}\right]$$

$$n_p = \exp\left[-\frac{m_p - \mu_p}{T}\right]$$

$$\mu_n + \mu_\nu = \mu_p + \mu_e$$

gives

$$\frac{n_n}{n_p} = \exp\left(-\frac{I}{T} + \frac{\mu_e - \mu_\nu}{T}\right)$$

1.3 MeV 

$$\frac{\mu_e}{T} \approx \frac{n_{e^-} - n_{e^+}}{n_{e^-} + n_{e^+}} \approx \eta_B \sim 10^{-10}$$

thus because $n_{e^-} - n_{e^+} - n_p = 0$
(neutrality)

and $n_e \sim n_\gamma$, electrons are relativistic.

We also assume μ_D to be small
(we could treat it as a parameter that is later fixed)

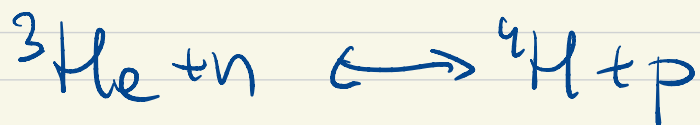
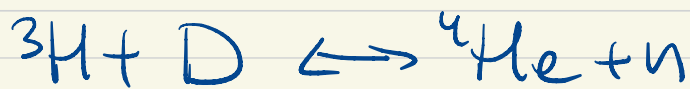
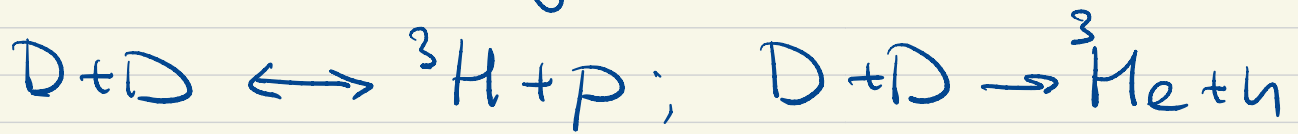
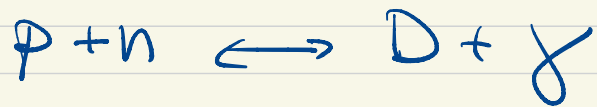
$$\frac{n_n}{n_p} = e^{-\frac{m_n - m_p}{T_*}} \approx \frac{1}{5}$$

Note that timescale (age of the universe) is $t_* \sim 1s$,

which is much shorter than lifetime of neutron (15 minutes)

Nuclear Reactions

Next step is to wait until the nuclear reactions become important:



Binding energy I for a given nucleus is given by

$$I = Zm_p + (A-Z)m_n - m$$

What is important, is the ratio

$$\frac{I}{A}: \quad {}^2\text{H} (D) \sim 1.1 \text{ MeV} \quad {}^4\text{He} \sim 7 \text{ MeV}$$

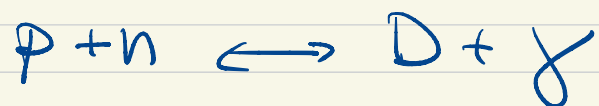
$${}^3\text{H} \sim 2.3 \text{ MeV} \quad {}^{12}\text{C} \sim 7.7 \text{ MeV}$$

Γ_{Fe} is the largest

The reactions active during BBN never produce ^{12}C , but they produce ^4He in which most of the neutrons end up being

First, however, deuterium has to be produced.

Let's estimate when reaction



gets shifted to the right

For that, let us compute temperature at which abundance of neutrons and deuterium are equal.

F.c.f. the computation we did for CMB]

$$\frac{n_p n_n}{n_D} = \left(\frac{m_p T}{2\pi} \right)^{3/2} e^{-\frac{I_D}{T}} \quad]$$

$$n_p \sim \eta_B \cdot n_f \sim 0.24 T^3 \eta_B$$

gives $T_{ns} \sim 70 \text{ keV}$

- We assumed that neutrons are in the thermal equilibrium, however, they froze out already and some of them decayed.

In fact $t_{ns} \sim 4 \text{ min}$

$$\frac{n_n}{n_p} \sim e^{-\frac{T}{T_2}} \cdot e^{-\frac{t_{ns}}{\tau_n}} \approx \frac{1}{7}$$

\searrow
 $\frac{1}{5}$

${}^4\text{He}$ abundance (equilibrium)

Let's define abundances of nuclei

$$x_A = \frac{A n_A}{n_B}, \quad \sum x_A = 1$$

Since binding energy per nucleon (and per neutron) is locally the largest for ${}^4\text{He}$, and there are fewer neutrons than protons production of ${}^4\text{He}$ will continue until all neutrons are in ${}^4\text{He}$:

$$n_{{}^4\text{He}} = \frac{1}{2} n_n$$

$$x_4 = \frac{4 n_{{}^4\text{He}}}{n_p + n_n} = \frac{2 \cdot \frac{1}{7}}{1 + \frac{1}{7}} \approx 25\%$$

Let's check it more carefully:

chemical equilibrium reads

$$\mu_A = \mu_p \cdot z + \mu_n (A - z), \text{ then}$$

$$n_A \approx n_p^z n_n^{A-z} \cdot \left(\frac{1}{m_p T} \right)^{\frac{3}{2}(A-1)} e^{\frac{I_A}{T}}$$

$$n_B \approx n_B T^3 \approx n_p$$

$$X_n \approx X_{He}^{1/2} n_B^{-3/2} \left(\frac{T}{m_p} \right)^{-9/4} e^{-\frac{I_{He}}{2T}}$$

$$X_A \approx \left[n_B \left(\frac{T}{m_p} \right)^{3/2} \right]^{\frac{3}{2}z - \frac{1}{2}A - 1} e^{I_A - I_{He} \frac{(A-z)}{2}}$$

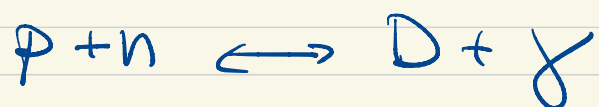
$$\sim e^{\frac{I_A}{A-z} - \frac{I_{He}}{4-z}}$$

Here what matters is binding energy per n !

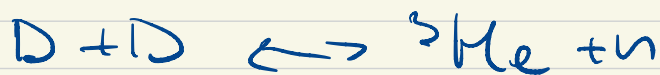
If we put I_D into this formula we get $x_D \sim e^{-120}$...

In reality the primordial fraction of Deuterium is not as small, because the reaction rates are quite and they freeze out before all Deuterium, and other elements burn.

Abundance of Deuterium



(production)



(destruction
- burning)

$$\frac{dn_D}{dt} + 3Hn_D = -\langle \sigma v n_D \rangle n_D$$

$$\sigma v \approx 3 \cdot 10^{-17} \frac{\text{cm}^3}{\text{s}} \quad \text{at } T > T_{\text{us}}$$

As usual:

$$H = \frac{T^2}{M_0}$$

$$n_P \sim \eta_B T^3$$

$\Gamma \approx H$ condition reads

$$\frac{T^2}{M_0} = \sigma v n_D \Rightarrow$$

$$\Rightarrow \frac{n_D}{n_P} = \frac{1}{\sigma v \eta_B M_0 T_{us}} \approx 10^{-5}$$

[More precise computations in Rubakov's Book]

- $10^{-5} \gg e^{-120}$!

- $10^{-5} \ll 1$

- $\frac{n_D}{n_P} \sim \frac{1}{\eta_B} \rightarrow$ sensitive to it !