Lecture 8

Big-Bang Nucleosynthesis

- Review (and future plan)
- BBN
  - → general picture
  - → ^4He abundance (equilibrium)
  - → D abundance (finite reactions rate)

Review

CMB:

Saha's equation

$$\frac{n_e n_p}{n_H} = \left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-\frac{I}{T}}$$

$$\frac{\partial n}{\partial t} + 3H \frac{\partial n}{\partial p} = \langle \sigma n v \rangle n$$

$[N \to n]$

$e\gamma \leftrightarrow e\gamma$ process freezes out

$$T = \frac{I}{2\cdot 27} = 0.25\,eV$$ [electrons non-rel.]

Neutrinos

$e\nu \leftrightarrow e\nu$ process freezes out

$$T_*^\nu = \left(G_F^2 M_0\right)^{-1/3} \sim 2\,MeV$$ [neutrinos rel.]

e^+e^- annihilation heats up Neutrinos

$$T_{\nu,0} \approx 2\,K \qquad (T_{\gamma,0} = 2.6\,K)$$

$\rightsquigarrow$ cosmological bound on m_ν

Future Plans:

- Baryogenesis and Dark Matter
- Theory of perturbations (5 lectures)

Rubakov, Gorbunov II

$\rightsquigarrow$ CMB

$\rightsquigarrow$ Large scale structure

$\rightsquigarrow$ Inflation

BBN

- BBN is the process of formation of ligh nuclei (H, He, D, Li...) from p and n in the primordial plasma. It occures at $T \sim 1 - 0.1$ MeV. Atoms form much later (during recombination) and all heavier elements even later in stars.

- As a result of BBN we get 75% of H, 25% of He and $\sim 2 \cdot 10^{-5}$ of D. These abundances are measured and since they depend significantly on various parameters N_{eff}, η_B, et.c., these parameters get constrained.

Neutron - proton ratio, freeze - out of neutrons

Remember that $m_p \sim m_n \sim 1$ GeV, while $m_n - m_p \simeq 1.3$ MeV.

Let us check when the reaction

$$p + e \longleftrightarrow n + \nu_e$$

(also possible cross-channels)

will go out of equilibrium

It is a weak interaction, so T^* will be close to that of neutrinos

$$T_\nu^* \sim \left(\frac{M_0}{G_F^2}\right)^{1/3} \sim 2 \text{ MeV}$$

[A more precise calculation gives

$$T_n^* \sim 0.8 \text{ MeV}]$$

Both are comparable to $m_n - m_p$
[It is an important coincidence!]

Let us compute the abundances, assuming thermal equilibrium (ν's just froze out, but they are still in the thermal eq.)

We have around $n, p, e^{\pm}, \nu$'s

$$n_n \approx \exp\left[-\frac{m_n - \mu_n}{T}\right]$$

$$n_p = \exp\left[-\frac{m_p - \mu_p}{T}\right]$$

$$\mu_n + \mu_\nu = \mu_p + \mu_e$$

gives

1.3 MeV

$$\frac{n_n}{n_p} = \exp\left(-\frac{I}{T} + \frac{\mu_e - \mu_\nu}{T}\right)$$

$$\frac{\mu_e}{T} \simeq \frac{n_{e^-} - n_{e^+}}{n_{e^-} + n_{e^+}} \simeq \eta_B \sim 10^{-10}$$

this because $n_{e^-} - n_{e^+} - n_p = 0$
(neutrality)

and $n_e \sim n_\gamma$, electrons are relativistic.

We also assume μ_ν to be small (we could treat it as a parameter that is later fixed)

$$\frac{n_n}{n_p} \simeq e^{-\frac{m_n - m_p}{T_*}} \simeq \frac{1}{5}$$

Note that timescale (age of the universe) is $t_* \sim 1s$, which is much shorter than lifetime of neutron (15 minutes)

Nuclear Reactions

Next step is to wait until the nuclear reactions become important:

$$p + n \longleftrightarrow D + \gamma$$

$$D + D \longleftrightarrow {}^3H + p; \quad D + D \rightarrow {}^3He + n$$

$$ {}^3H + D \longleftrightarrow {}^4He + n$$

$$ {}^3He + n \longleftrightarrow {}^4H + p$$

Binding energy I for a given nucleus is given by

$$I = Z m_p + (A - Z) m_n - m$$

What is important, is the ratio

$\frac{I}{A}$: ^2H (D) ~ 1.1 MeV $\qquad$ $^4He \sim 7$ MeV

$^3H \sim 2.3$ MeV $\qquad$ $^{12}C \sim 7.7$ MeV

I_{Fe} is the largest.

The reactions active during BBN never produce ^{12}C, but they produce ^4He in which most of the neutrons end up being

First, however, deiterium has to be produced.

Let's estimate when reaction

$$p + n \longleftrightarrow D + \gamma$$

gets shifted to the right

For that, let us compute temperature at which abundance of neutrons and deiterium are equal.

[c.f. the computation we did for CMB]

$$\frac{n_p n_n}{n_D} = \left(\frac{m_p T}{2\pi}\right)^{3/2} e^{-\frac{I_D}{T}}$$

$$n_p \sim \eta_B \cdot n_\gamma \sim 0.24 T^3 \eta_B$$

gives $T_{NS} \sim 70$ keV

- We assumed that neutrons are in the termal equilibrium, however, they froze out already and some of them decayed.

In fact $t_{NS} \sim 4$ min

$$\frac{n_n}{n_p} \sim \underbrace{e^{-\frac{I}{T_*}}}_{\frac{1}{5}} \cdot e^{-\frac{t_{NS}}{\tau_n}} \simeq \frac{1}{7}$$

^4He abundance (equilibrium)

Let's define abundances of nuclei

$$x_A = \frac{A\, n_A}{n_B}, \quad \sum x_A = 1$$

Since binding energy per nucleon (and per neutron) is locally the largest for ^4He, and there are fewer neutrons than protons production of ^4He will continue until all neutrons are in it:

$$n_{^4He} \simeq \frac{1}{2} n_n$$

$$x_4 = \frac{4\, n_{^4He}}{n_p + n_n} = \frac{2 \cdot \frac{1}{7}}{1 + \frac{1}{7}} \simeq 25\%$$

Let's check it more carefully:

chemical equilibrium reads

$$\mu_A = \mu_p \cdot Z + \mu_n (A - Z) \;, \text{ then}$$

$$n_A \simeq n_p^{Z}\, n_n^{A-Z} \cdot \left(\frac{1}{m_p T}\right)^{\frac{3}{2}(A-1)} e^{\frac{I_A}{T}}$$

$$n_B \simeq \eta_B T^3 \simeq n_p$$

$$X_n \simeq X_{^4He}^{1/2}\, \eta_B^{-3/2} \left(\frac{T}{m_p}\right)^{-9/4} e^{-\frac{I_{^4He}}{2T}}$$

$$X_A \simeq \left[\eta_B \left(\frac{T}{m_p}\right)^{3/2}\right]^{\frac{3}{2}Z - \frac{1}{2}A - 1} e^{I_A - I_{^4He}\frac{(A-Z)}{2}} \sim$$

$$\sim e^{\frac{I_A}{A-Z} - \frac{I_{^4He}}{4-2}}$$

Here what matters is binding energy per n!

If we put I_D into this formula we get $x_D \sim e^{-120}$ ....

In reality the primordial fraction of Deiterium is not as small, because the reaction rates are finite and they freeze out before all Deiterium, and other elements burn.

Abundance of Deiterium

$p + n \longleftrightarrow D + \gamma$ (production)

$D + D \longleftrightarrow {}^3He + n$
$D + D \longleftrightarrow T + p$
(distruction - burning)

$$\frac{dn_D}{dt} + 3Hn_D = -\langle \sigma v \, n_D \rangle n_D$$

$$\sigma v \simeq 3 \cdot 10^{-17} \frac{cm^3}{s} \quad \text{at } T > T_{NS}$$

As usual:

$$H \simeq \frac{T^2}{M_0} \qquad n_p \sim \eta_B T^3$$

$\Gamma \simeq H$ condition reads

$$\frac{T^2}{M_0} \simeq \sigma v n_D \Rightarrow$$

$$\Rightarrow \frac{n_D}{n_p} \simeq \frac{1}{\sigma v \eta_B M_0 T_{NS}} \simeq 10^{-5}$$

[More precise computations in Rubakov's Book]

- $10^{-5} \gg e^{-120}$!
- $10^{-5} \ll 1$
- $\frac{n_D}{n_p} \sim \frac{1}{\eta_B} \rightarrow$ sensitive to it!